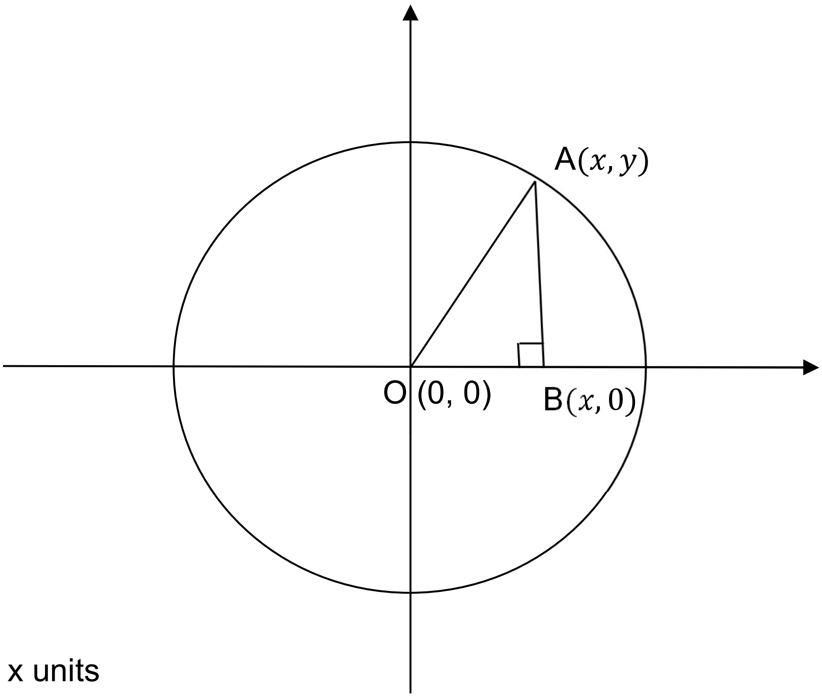
EQUATION OF A CIRCLE

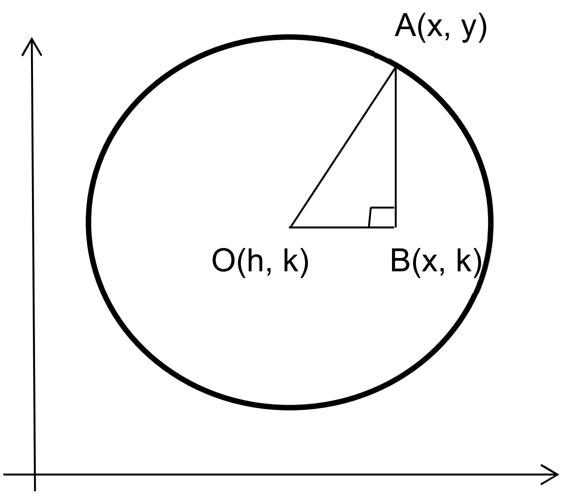
Let A (x, y) be any point on the circle and O(0,0) be the center of the circle.



OB = x units
AB = y units

OA = $\sqrt{x^2 + y^2}$ units

Let A(x, y) be any point on the circle, O (h, k) be the center of the circle and r be the radius of the circle.



OB = x – h units
AB = y – k units

r = OA = $\sqrt{(x-h)^2 + (y-k)^2}$

$$r^2 = (x-h)^2 + (y-k)^2$$

or

$$r^2 = x^2 + y^2 - 2xh - 2xy + h^2 + k^2$$

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Example 1: Find the equation of the circle with center C (3, 5) and radius 4.

Solution:
$$(x-3)^2 + (y-5)^2 = 4^2$$
$$x^2 - 6x + 9 + y^2 - 10y + 25 = 16$$
$$x^2 + y^2 - 6x - 10y + 18 = 0$$

Example 2: Find the center and radius of the equation $(x-2)^2 + (y+5)^2 = 21$ and sketch the graph.

Solution: center = (2, -5) and radius = $\sqrt{21}$

Example 3: Find the equation of a circle that has a diameter with the Endpoints given by the points A(-2, 3) and B(4, 7).

Solution: Midpoint of $AB = \left(\frac{-2+4}{2}, \frac{3+7}{2}\right)$
$$= (1, 5)$$

Radius $= \sqrt{13}$

$$(x-1)^2 + (y-5)^2 = 13$$

<u>Exercise 1</u>

1. Find the equation of the circle with
   a) Center (0, 0), radius 3,
   b) Center $\left(\frac{1}{2}, -\frac{2}{5}\right)$, radius $\frac{3}{2}$
   c) Center (4, -1), passing through (-2, 0)

2. Find the equation of the circle with the points (-3, 2) and (3, -2) as ends of its diameter.

Example 4: Given the equation of a circle is $x^2 + y^2 - 8x + 4y - 8 = 0$, Find its center and radius.

Solution:

$$x^2 + y^2 - 8x + 4y - 8 = 0$$
$$x^2 - 8x + y^2 + 4y - 8 = 0$$
$$(x-4)^2 + (y+2)^2 - 20 - 8 = 0$$
$$(x-4)^2 + (y+2)^2 - 28 = 0$$

$$radius = \sqrt{28}$$

$$center\ of\ circle = (4, -2)$$

Example 5: The line $y = 2x + 5$ cuts the circle $x^2 + y^2 = 10$ at two Points A and B. Find

(a) The coordinates of A and B

(b) The equation of the perpendicular bisector of AB and show that it passes through the center of the circle.

(c) The coordinates of the points where the perpendicular bisector cuts the circle.

Solution: (a) $substituting\ y = 2x + 5\ into\ x^2 + y^2 = 10 - - - - - - - - (1)$

$$x^2 + (2x+5)^2 = 10$$
$$x^2 + 4x^2 + 20x + 25 - 10 = 0$$
$$5x^2 + 20x + 15 = 0$$
$$x^2 + 4x + 3 = 0$$
$$(x+1)(x+3) = 0$$
$$y = -1\ or\ -3$$
$$y = 3\ or -1$$
$$the\ coordinates\ are\ (-1, 3) and\ (-3, -1)$$

(b) Midpoints of AB is (-2, 1)

Gradient of the perpendicular bisector of $AB = -\frac{1}{2}$

$$\frac{y-1}{x+2} = -\frac{1}{2}$$
$$2y = -x - - - - - - - - - (2)$$

Since the equation has no constant term, the perpendicular bisector of AB passes through the origin (0, 0), i.e. the center of the circle.

(c). Solving equations (1) and (2):

$$4y^2 + y^2 = 10$$
$$y^2 = 2$$
$$y = +\sqrt{2}\ or - \sqrt{2}\ ; x = -2\sqrt{2}\ or\ 2\sqrt{2}$$

Hence the coordinates of the points where the perpendicular bisector cuts the circles are $(-2\sqrt{2}, \sqrt{2}) and\ (2\sqrt{2}, -\sqrt{2})$.

Exercise 2

1. Find the center and the length of radius of each of the following circles,

   (a) $x^2 + y^2 = 49$
   (b) $2x^2 + 2y^2 = 5$
   (c) $4x^2 + 4y^2 - 6x + 10y = \frac{1}{2}$

2. Find the equation of the circle which passes through the points A(-2, 0) and B(5, 1) and whose center lies on the line $2x + y = 1$.