<u>Vectors</u>

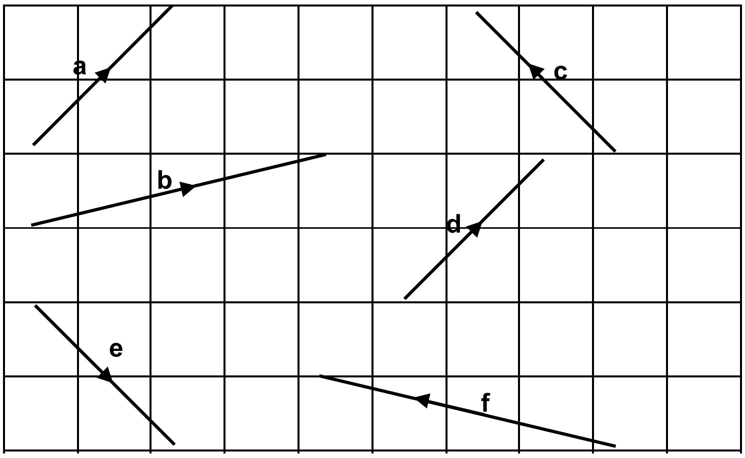
Vectors have <u>magnitude</u> and <u>direction</u>. [Scalars have only magnitude.]
Example of vectors: velocity, distance
Example of scalars: speed, displacement

Equal vectors are any 2 vectors that have the same magnitude and same direction.

Negative vector of $\boldsymbol{a}$, denoted by $-\boldsymbol{a}$, it has the same magnitude as $\boldsymbol{a}$ but in the opposite direction.

Zero vector (or null vector) denoted by $\mathbf{0}$, it has zero magnitude.

(Note: Vectors are written in bold, however in writing, we "bold" these vectors using a wave symbol below it. Example $\boldsymbol{a}$, is written like this "$\underset{\sim}{a}$".



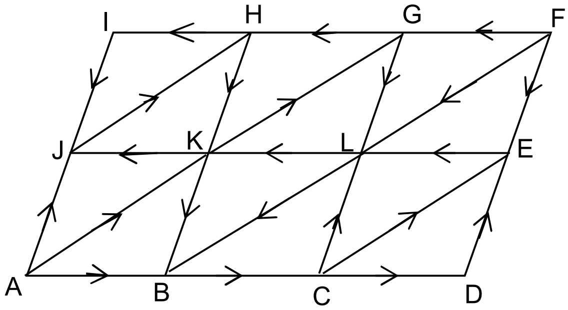
<u>Exercise 1</u>

1. (a) Name all the equal vectors of

(i) $\overrightarrow{IJ}$ (ii) $\overrightarrow{AJ}$ (iii) $\overrightarrow{HI}$
(iv) $\overrightarrow{BC}$ (v) $\overrightarrow{AK}$ (vi) $\overrightarrow{LB}$

(b) Name all the negative vector of

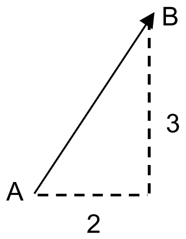
(i) $\overrightarrow{JH}$ (ii) $\overrightarrow{AB}$ (iii) $\overrightarrow{AJ}$

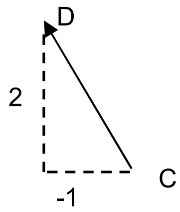


Column Vector

A vector can be written in a column vector.

Example:
$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, means 2 units parallel to the x −axis and 3 units parallel to the y −axis. $\overrightarrow{CD} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, means -1 units parallel to the x −axis and 2 units parallel to the y −axis.

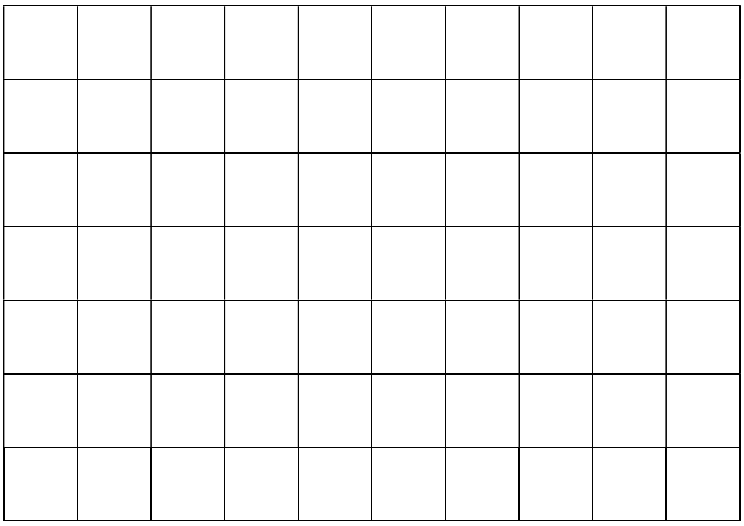




<u>Exercise 2</u>

1. Can you try drawing the following vectors?

(a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, (b) $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, (c) $\overrightarrow{EF} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$,

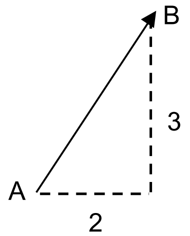


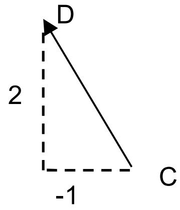
2. Can you write their negative vectors using column vectors?

(a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, (b) $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, (c) $\overrightarrow{EF} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$,

Magnitude

If $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, then the magnitude of $\overrightarrow{AB}$ denoted by $|\overrightarrow{AB}|$, is equal to $\sqrt{x^2 + y^2}$





$|\overrightarrow{AB}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

$|\overrightarrow{CD}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

<u>Exercise 3</u>

1. Find the magnitude of each of the following vectors, leaving the answer in square root form where necessary.

(a) $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ (b) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} -6 \\ -2 \end{pmatrix}$ (d) $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

2. Given that $\boldsymbol{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} s \\ 0 \end{pmatrix}$, where s is positive, find the value of s such that $|\boldsymbol{a}| = |\boldsymbol{b}|$.

3. Given that $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} p \\ -9 \end{pmatrix}$, find
   (a) $|\overrightarrow{AB}|$, giving your answer correct to 2 significant figures.
   (b) the positive value of *p* if $|\overrightarrow{CD}| = 3|\overrightarrow{AB}|$.

Scalar Vectors

| k**a** | = | k | . | **a** | where k is a scalar
If k > 0 $\Rightarrow$ k**a** and **a** have the same direction
If k < 0 $\Rightarrow$ k**a** and **a** have opposite directions

<u>Example:</u>

If $\overrightarrow{PQ} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$, find $\frac{1}{2}\overrightarrow{PQ}$ and $\frac{1}{2}\overrightarrow{QP}$.

<u>Solution:</u>

$$\frac{1}{2}\overrightarrow{PQ} = \frac{1}{2}\begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$\frac{1}{2}\overrightarrow{QP} = \frac{1}{2}\left(-\overrightarrow{PQ}\right) \left[\text{since } \overrightarrow{QP} = -\overrightarrow{PQ}\right]$$

$$= -\frac{1}{2}\begin{pmatrix} -4 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

<u>Exercise 4</u>

1. If $\underset{\sim}{a} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ and $\underset{\sim}{c} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$, find the column vector for each of the following:

(a) $3\boldsymbol{a} - 2\boldsymbol{c}$ (b) $2\boldsymbol{a} - \frac{1}{3}\boldsymbol{c}$ (c) $2\boldsymbol{a} + \boldsymbol{b} - 3\boldsymbol{c}$ (d) $4\boldsymbol{a} + \boldsymbol{b} - \frac{1}{2}\boldsymbol{c}$

2. If $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -12 \\ 3 \end{pmatrix}$ and $\overrightarrow{EF} = \begin{pmatrix} 5 \\ -15 \end{pmatrix}$, evaluate the following:

(a) $\frac{1}{3}\overrightarrow{AB}$ (b) $\overrightarrow{AB} + 2\overrightarrow{CD}$ (c) $3\overrightarrow{CD} + 2\overrightarrow{EF} - \overrightarrow{AB}$ (d) $\frac{2}{3}\overrightarrow{AB} - \frac{1}{3}\overrightarrow{CD} - \frac{3}{5}EF$

Parallel Vectors

When two vectors can be expressed as a scalar vector, we write $\boldsymbol{a} = k\boldsymbol{b}$ where k is a scalar, it also means that

(i) $\boldsymbol{a}$ is parallel to $\boldsymbol{b}$

(ii) $|\boldsymbol{a}| = k\,|\boldsymbol{b}|$

<u>Example:</u>

$\boldsymbol{a} = \binom{2}{3}$ and $\boldsymbol{b} = \binom{4}{6}$

We can write,

$$\boldsymbol{b} = \binom{4}{6} = 2\binom{2}{3} = 2\boldsymbol{a}$$
$$\therefore \boldsymbol{b} = 2\boldsymbol{a}$$

We can note 2 things about $\boldsymbol{b} = 2\boldsymbol{a}$,

Firstly, $\boldsymbol{b}$ is parallel to $\boldsymbol{a}$ and secondly, the magnitude of $\boldsymbol{b}$ is 2 times of $\boldsymbol{a}$

<u>Example:</u>

Given that $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} u \\ 24 \end{pmatrix}$ are parallel vectors, find the value of u.

<u>*Solution:*</u>

$\begin{pmatrix} 3 \\ -6 \end{pmatrix} = k\begin{pmatrix} u \\ 24 \end{pmatrix}$

$3 = ku$

$-6 = 24k$

$k = -\frac{1}{4}$

$3 = -\frac{1}{4}u$

$\therefore u = -12$

Alternatively,

$\frac{3}{-6} = \frac{u}{24}$

$u = -\frac{3}{6} \times 24$

$= -12$

<u>Exercise 5</u>

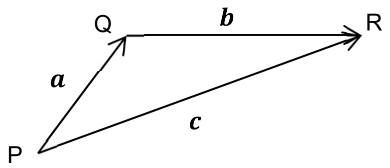
1. State which of the following pairs of vectors are parallel, if they are, write them in the form $\boldsymbol{a} = k\boldsymbol{b}$

(a) $\begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ (c) $\begin{pmatrix} 10 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

(d) $\begin{pmatrix} 10 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ (e) $\begin{pmatrix} -3 \\ -21 \end{pmatrix}, \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ (f) $\begin{pmatrix} 6 \\ 2 \end{pmatrix}, \begin{pmatrix} 12 \\ 4 \end{pmatrix}$

2. Given that $\begin{pmatrix} 8 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ p \end{pmatrix}$ are parallel vectors, find the value of *p*.

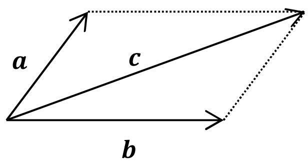
Addition of Vectors
Using the triangular law of vector addition



$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$ or $\overrightarrow{\boldsymbol{PQ}} + \overrightarrow{\boldsymbol{QR}} = \overrightarrow{\boldsymbol{PR}}$

Note: Q is a common point that connects the vectors together

OR
Using the parallelogram law of vector addition



Subtraction of Vectors
Subtraction of vectors can be treated as addition of negative vectors

For example

$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$$

$$\begin{aligned} \overrightarrow{\boldsymbol{PQ}} - \overrightarrow{\boldsymbol{RQ}} &= \overrightarrow{\boldsymbol{PQ}} + (-\overrightarrow{\boldsymbol{RQ}}) \\ &= \overrightarrow{\boldsymbol{PQ}} + \overrightarrow{\boldsymbol{QR}} \\ &= \overrightarrow{\boldsymbol{PR}} \end{aligned}$$

Exercise 6

1. Express in terms of a single vector the sum of the vectors.

(a) $\overrightarrow{\boldsymbol{VZ}} + \overrightarrow{\boldsymbol{ZX}} =$ (b) $\overrightarrow{\boldsymbol{YW}} + \overrightarrow{\boldsymbol{WX}} =$

(c) $\overrightarrow{\boldsymbol{YZ}} + \overrightarrow{\boldsymbol{ZX}} =$ (d) $\overrightarrow{\boldsymbol{YW}} + \overrightarrow{\boldsymbol{WZ}}$

(e) $\overrightarrow{\boldsymbol{VW}} + \overrightarrow{\boldsymbol{YW}} + \overrightarrow{\boldsymbol{WX}} =$ (f) $\overrightarrow{\boldsymbol{XW}} + \overrightarrow{\boldsymbol{WZ}} + \overrightarrow{\boldsymbol{ZV}} + \overrightarrow{\boldsymbol{VY}} =$

2. Simplify the following

(a) $\overrightarrow{\boldsymbol{PS}} + \overrightarrow{\boldsymbol{SR}} =$ (b) $\overrightarrow{\boldsymbol{QS}} + \overrightarrow{\boldsymbol{PQ}}$

(c) $\overrightarrow{\boldsymbol{PR}} + \overrightarrow{\boldsymbol{RQ}} + \overrightarrow{\boldsymbol{QS}}$ (d) $\overrightarrow{\boldsymbol{PQ}} + \overrightarrow{\boldsymbol{QR}} + \overrightarrow{\boldsymbol{RP}} =$

(e) $\overrightarrow{\boldsymbol{PR}} - \overrightarrow{\boldsymbol{SR}} =$ (f) $\overrightarrow{\boldsymbol{RQ}} + \overrightarrow{\boldsymbol{QS}} - \overrightarrow{\boldsymbol{PS}} =$

3. Find for each of the following equations a vector which can replace u.

(a) $\overrightarrow{\boldsymbol{DX}} + \boldsymbol{u} = \mathbf{0}$ b) $\overrightarrow{\boldsymbol{DA}} + \overrightarrow{\boldsymbol{AB}} + \boldsymbol{u} = \mathbf{0}$

(c) $\overrightarrow{\boldsymbol{AD}} + \overrightarrow{\boldsymbol{DC}} + (-\overrightarrow{\boldsymbol{XC}}) = \boldsymbol{u}$ d) $\overrightarrow{\boldsymbol{AX}} + (-\overrightarrow{\boldsymbol{DX}}) = \boldsymbol{u}$

(e) $\overrightarrow{\boldsymbol{AD}} + (-\overrightarrow{\boldsymbol{CD}}) = \boldsymbol{u}$ f) $\overrightarrow{\boldsymbol{AB}} + \overrightarrow{\boldsymbol{BC}} + (-\overrightarrow{\boldsymbol{AC}}) = \boldsymbol{u}$

<u>Position Vectors</u>

When given a coordinate $A(2,3)$ means that it can be represented in a position vector $\overrightarrow{\boldsymbol{OA}} = \begin{pmatrix} \mathbf{2} \\ \mathbf{3} \end{pmatrix}$

<u>Example</u>

Given the coordinate of $A(2,4)$ and $B\ (-1,3)$, find $\overrightarrow{\boldsymbol{AB}}$

<u>Solution</u>

We are given $\overrightarrow{\boldsymbol{OA}} = \begin{pmatrix} \mathbf{2} \\ \mathbf{4} \end{pmatrix}$ and $\overrightarrow{\boldsymbol{OB}} = \begin{pmatrix} \mathbf{-1} \\ \mathbf{3} \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{\boldsymbol{AB}} &= \overrightarrow{\boldsymbol{AO}} + \overrightarrow{\boldsymbol{OB}} \\ &= -\begin{pmatrix} \mathbf{2} \\ \mathbf{4} \end{pmatrix} + \begin{pmatrix} \mathbf{-1} \\ \mathbf{3} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{-2-1} \\ \mathbf{-4+3} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{-3} \\ \mathbf{-1} \end{pmatrix} \end{aligned}$$

<u>Example</u>

Find the coordinates of D if $\overrightarrow{\boldsymbol{AD}} = \begin{pmatrix} \mathbf{4} \\ \mathbf{-2} \end{pmatrix}$ and $A\ (-1,1)$

<u>Solution</u>

$$\begin{aligned} \overrightarrow{\boldsymbol{OD}} &= \overrightarrow{\boldsymbol{OA}} + \overrightarrow{\boldsymbol{AD}} \\ &= \begin{pmatrix} \mathbf{-1} \\ \mathbf{1} \end{pmatrix} + \begin{pmatrix} \mathbf{4} \\ \mathbf{-2} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{3} \\ \mathbf{-1} \end{pmatrix} \end{aligned}$$

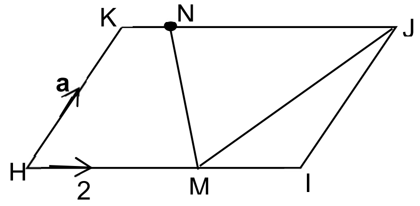
Therefore the coordinates of D is $(3,-1)$

Now let us apply what you have learn to solve more complicated questions!

Example 1

In the diagram, HIJK is a parallelogram. M is the mid-point of HI and N is on KJ such that KJ = 3 KN. Given that $\overrightarrow{HK} = \underset{\sim}{a}$ and $\overrightarrow{HM} = 2\underset{\sim}{b}$, express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(a) $\overrightarrow{MJ}$ b) $\overrightarrow{JN}$ c) $\overrightarrow{NM}$



Example 2

ABCD is a parallelogram with M as the mid-point of BC. If $\overrightarrow{AB} = \underset{\sim}{u}$ and $\overrightarrow{AD} = \underset{\sim}{v}$, express in terms of $\underset{\sim}{u}$ and/or $\underset{\sim}{v}$

(a) $\overrightarrow{CM}$ (b) $\overrightarrow{DB}$ (c) $\overrightarrow{AM}$ (d) $\overrightarrow{MD}$