


Algebraic Expansion & Factorisation

Expansion



$$(a + b)(c + d) = ac + ad + bc + bd$$

Common Errors:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^2 = a^2 + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)^2 = a^2 - b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

Examples:

1. $(3x + 4)^2$

$$= (3x)^2 + 2(3x)(4) + 4^2$$

$$= 9x^2 + 24x + 16$$

2. $(5x - 6)^2$

$$= (5x)^2 - 2(5x)(6) + 6^2$$

$$= 25x^2 - 60x + 36$$

3. $(2x + 1)(2x - 1)$

$$= (2x)^2 - 1^2$$

$$= 4x^2 - 1$$

4. $(x + 2)(x - 5)$

$$= x^2 - 5x + 2x - 10$$

$$= x^2 - 3x - 10$$

5. $(3x^2 - 5x + 2)(4x - 3)$

$$= 12x^3 - 9x^2 - 20x^2 + 15x + 8x - 6$$

$$= 12x^3 - 29x^2 + 23x - 6$$

Exercise 1:

Expand the following:

1. $(a + 7)^2$

2. $(m + 6)^2$

3. $(a - 2)^2$

4. $(2n - 5)^2$

5. $(1 - 6x)^2$

6. $(x - \frac{3}{4})^2$

7. $(2 + 7x)(2 - 7x)$

8. $(x^2 + 7y)(x^2 - 7y)$

9. $(y + 2)(y - 8)$

10. $(m - 5)(4m + 3)$

11. $(x^2 + 2xy + y^2)(x + 2y)$

12. $(3y^2 - 5y + 4)(3y - 7)$

Factorisation

1. Take out common factor

e.g. $ac + ad = a(c + d)$

Examples:

$$\begin{aligned} 1. \quad & 49x + 7 \\ & = 7(7x + 1) \end{aligned}$$

$$\begin{aligned} 2. \quad & 4xy + 18xyz \\ & = 2xy(2 + 9z) \end{aligned}$$

$$\begin{aligned} 3. \quad & 2xy^2 + 3x^2y - 5x^2y^2 \\ & = xy(2y + 3x - 5xy) \end{aligned}$$

Exercise 2:

$$1. \quad 24x + 36$$

$$2. \quad 3ac + 9ab$$

$$3. \quad 5x + hx$$

$$4. \quad 5a^2x + 10ay$$

$$5. \quad 6ax^2 + 2a^2x$$

$$6. \quad 5a^3b^2c + 15a^2b^3c^2$$

$$7. \quad a^3b^2 - a^2b^3 + ab$$

$$8. \quad acx^2 - bcx^4 + abx^3$$

2. Grouping

$$\begin{aligned} \text{e.g. } & ab + ac + db + dc \\ & = a(b + c) + d(b + c) \\ & = (a + d)(b + c) \end{aligned}$$

$$\begin{aligned} \text{e.g. } & ab + ac - db - dc \\ & = a(b + c) - d(b + c) \\ & = (a - d)(b + c) \end{aligned}$$

Examples:

$$\begin{aligned} 1. \quad & pq + pr - q^2 - qr \\ & = p(q + r) - q(q + r) \\ & = (p - q)(q + r) \end{aligned}$$

Exercise 3:

$$1. \quad xa + 2xz + 4a + 8z$$

$$2. \quad x^2 + xc + yc + xy$$

$$3. \quad xy - xa - yd + ad$$

$$4. \quad 3y^2 - yz + 9y - 3z$$

$$5. \quad a + b - xa - xb$$

$$6. \quad cx - cy - y^2 + xy$$

3. Perfect Squares & Difference of Squares

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

Examples:

1. $9x^2 + 24x + 16$

$$= (3x)^2 + 2(3x)(4) + 4^2$$

$$= (3x + 4)^2$$

2. $25x^2 - 60x + 36$

$$= (5x)^2 - 2(5x)(6) + 6^2$$

$$= (5x - 6)^2$$

3. $4x^2 - 1$

$$= (2x)^2 - (1)^2$$

$$= (2x + 1)(2x - 1)$$

4. If $p^2 - q^2 = 48$ and $p + q = 6$, find the value of $(p - q)^2$,

$$p^2 - q^2 = 48$$

$$(p + q)(p - q) = 48$$

$$6(p - q) = 48$$

$$(p - q) = 8$$

Therefore, $(p - q)^2 = 8^2 = 64$

Exercise 4:

1. $a^2 - 81$

2. $49x^4 - 1$

3. $4x^2 + 8x + 4$

4. $36c^2 - 9d^2e^2$

5. $9x^2y^2 - 12xy + 4$

6. $169 - 4x^2$

7. $16y^4 - 1$

8. $x^4 + 6x^2 + 9$

9. $\frac{25}{4}m^2 - \frac{5}{2}mn + \frac{1}{4}n^2$

10. $4(x + y)^2 - 9(x - y)^2$

11. $4(x - 5b)^2 - (2x - b)^2$

12. $16x^2 + 8nx + n^2$

13. $\frac{1}{4}m^2 + my + y^2$

14. $9a^2b^2 - 42ab + 49$

4. Quadratic Factorisation (Trial and Error)

$$x^2 + 2x - 3 = (x + 3)(x - 1)$$

x	3		3x
x	-1		-x
x^2	-3		$2x$

Examples:

1. $x^2 - 3x - 10 = (x + 2)(x - 5)$

x	2		2x
x	-5		-5x
x^2	-10		$-3x$

2. $2x^2 - 3xy + y^2 - 2x + y$
 $= (2x - y)(x - y) - (2x - y)$
 $= (2x - y)(x - y - 1)$

2x	-y		-2xy
x	-y		-xy
$2x^2$	y^2		$-3xy$

Exercise 5:

1. $6a^2 - 7a - 20$

2. $4p^2 - 8p + 3$

3. $12p^2 + 14p - 40$

4. $5x^2 + 13xy + 6y^2$

5. $5p^2 - 7p - 6$

6. $4b^2 - 7b + 3$

7. $6x^2 - 19x - 20$

8. $5x^2y^2 + 7xy - 6$

9. $12x^2b^2 - 38xb - 40$

10. $20x^2 + 52x + 24$

Solve Quadratic Equations by factorisation

Solve $x^2 + 2x - 3 = 0$

$$x^2 + 2x - 3 = (x+3)(x-1) = 0$$

either $(x+3) = 0$ or $(x-1) = 0$

$$x = -3 \text{ or } x = 1$$

Example:

1. $(3x - 4)^2 = 81$

$$(3x)^2 - 2(3x)(4) + (4)^2 = 81$$

$$9x^2 - 24x + 16 = 81$$

$$9x^2 - 24x - 65 = 0$$

$$(3x - 13)(3x + 5) = 0$$

either $(3x - 13) = 0$ or $(3x + 5) = 0$

$$x = \frac{13}{3} = 4 \frac{1}{3} \quad \text{or} \quad x = -\frac{5}{3} = -1 \frac{2}{3}$$

Exercise 6

Solve the following equations.

1. $3x^2 = 11x$

2. $6x^2 - 19x - 20 = 0$

3. $(x + 5)^2 = 4$

4. $4x^2 - 7x + 3 = 0$

5. $6x^2 = 7x + 20$

6. $5x^2 + 13x + 6 = 0$

7. $x^2 + 6x + 9 = 0$

8. $4x^2 = 8x - 3$

9. $x(2x + 7) = 4$

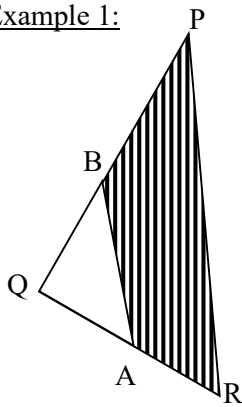
10. $7x^3 - 11x^2 = 30x$

11. $(x - 2)(x + 2) = 12$

12. $(x - 2)^2 = 9(x - 2)$

Quadratic Equations & Problem Sum

Example 1:



Given that

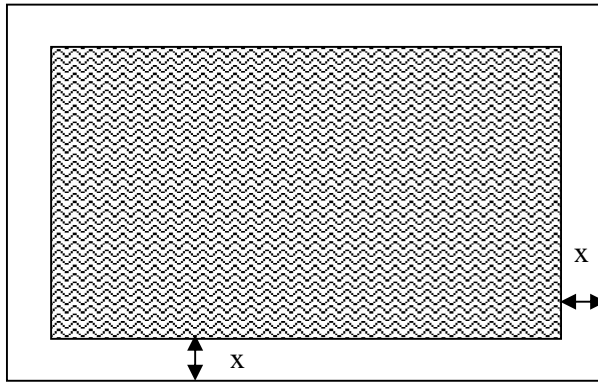
$$PQ = 5 \text{ cm}, QR = 7 \text{ cm}, \angle PQR = 90^\circ$$

$$PB = QA = x \text{ cm}$$

$$\text{Area of PBAR} = 14.5 \text{ cm}^2$$

Form a quadratic equation and solve for x.

Example 2:



The diagram shows a field of $L = 100$ m, $B = 50$ m, surrounded by a running track. The track has a constant width of x m. The area covered by the track is 775 m². Form a quadratic equation and find the width of track.